Geometric Origin of the $\pi/2$ Phase Lag in Imaginary Quasi-Periodic Oscillations: A TVS23 Derivation

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Abstract

Bellavita et al. (2025) [1] report the discovery of imaginary quasi-periodic oscillations (QPOs) in NICER observations of the black hole X-ray binary MAXI J1820+070. These oscillations appear as narrow features with a large imaginary and small real part in the Fourier cross-spectrum, with a phase lag of exactly $\pi/2$ relative to the standard QPO components, and are undetected in the power spectrum. The authors fit the data with a multi-Lorentzian model but conclude that the physical origin of the $\pi/2$ phase lag remains unexplained and that the imaginary QPO "may be a new type of QPO."

We show that the $\pi/2$ phase lag is not a free parameter but a geometric inevitability of the Structured Vacuum Theory TVS23 [3], in which the quantum vacuum is a rigid elastic network V_{23} with 23 torsional channels per node: 19 transverse (electromagnetic) and 4 longitudinal (gravitational). The transverse channels carry standard QPOs; the longitudinal channels carry oscillations that are exactly $\pi/2$ out of phase with the transverse modes by the geometry of the dodecahedral lattice. These are the imaginary QPOs. The frequency decrease from ~ 5 Hz to ~ 1 Hz during the soft-to-hard transition is derived from the progressive decoupling of gravitational channels as the accretion rate decreases. The universality of the effect across MAXI J1820+070, MAXI J1348–630, and Cygnus X-1 [2] is a direct prediction of TVS23: the channel geometry is identical in every region of the vacuum network. All results are derived with zero free parameters.

Contents

1 The Observational Anomaly

Bellavita et al. [1] discover that MAXI J1820+070 exhibits a QPO component that:

1. Has a phase lag of $\Delta\phi = \pi/2$ relative to the standard broadband variability components.
2. Is invisible in the power spectrum (PS) but clearly detected in the imaginary part of the cross-spectrum (CS).
3. Appears only during the soft-to-hard state transition, with frequency decreasing from ~ 5 Hz to ~ 1 Hz as the source hardens.
4. Has an rms spectrum that increases with photon energy, similar to type-C QPOs.
5. Is universal: the same phenomenon appears in MAXI J1348–630 and Cygnus X-1 [2].

The key mathematical fact exploited by Bellavita et al. is that a variability component with $\Delta\phi = \pi/2$ has zero real part and maximum imaginary part in the cross-spectrum. When such a component overlaps in frequency with standard ($\Delta\phi \approx 0$) components, it is suppressed in the PS but visible in the imaginary CS.

The authors fit the data successfully with multi-Lorentzian functions and reproduce the coherence drop. However, they leave the origin of the $\pi/2$ phase lag unexplained, treating it as an empirical input rather than a derived quantity.

2 TVS23: The 23-Channel Vacuum Network

2.1 Structure

In TVS23 [3], the quantum vacuum is a rigid elastic lattice V_{23} whose unit cell is a regular dodecahedron with edge length $\lambda_0 = 2a_0$ (a_0 = Bohr radius). Each node carries 23 torsional channels partitioned as:

$$23 = 19_{\text{EM}} + 4_{\text{grav}} + 2_{\text{temporal}}. \tag{1}$$

The partition is fixed by the representation theory of A_5 (the rotation symmetry group of the icosahedron/dodecahedron):

- 19_{EM}: transverse channels, perpendicular to the propagation direction. These carry electromagnetic radiation.
- 4_{grav}: longitudinal channels, parallel to the propagation direction. These carry gravitational perturbations.
- 2_{temporal}: fixed by the Euler characteristic $\chi = V - E + F = 20 - 30 + 12 = 2$ of the dodecahedral cell.

2.2 Phase relationship between transverse and longitudinal modes

In any elastic medium, transverse and longitudinal waves propagating in the same direction are exactly $\pi/2$ out of phase in their displacement fields. This is a fundamental result of wave mechanics:

Theorem 1 ($\pi/2$ geometric phase lag)**.** *In the V_{23} dodecahedral elastic network, the torsional displacement field of the longitudinal (gravitational) channels is exactly $\pi/2$ out of phase with the torsional displacement field of the transverse (electromagnetic) channels, for any propagation direction and any frequency.*

Proof. Let the torsional field in the transverse channels be $\Xi_\perp(\mathbf{r}, t) = A\cos(kx - \omega t)$. By the elastic wave equations of V_{23} [4], the longitudinal channel field satisfies:

$$\partial_t \Xi_\parallel = v_\parallel\, \partial_x \Xi_\perp,$$

where $v_\parallel = c/\sqrt{3}$ is the longitudinal wave speed derived from the elastic tensor $C_{iiii} = 12/5$, $C_{ijij} = 4/5$ [4]. Integrating:

$$\Xi_\parallel = \frac{v_\parallel}{v_\perp} A\sin(kx - \omega t) = \frac{1}{\sqrt{3}} A\sin(kx - \omega t).$$

The sine function is exactly $\pi/2$ out of phase with the cosine. The ratio of longitudinal to transverse amplitude is $1/\sqrt{3}$, consistent with Poisson ratio $\nu = 1/4$ (exact, from $v_\perp/c = 1/\sqrt{3}$). □ □

Corollary 1. *The imaginary QPO discovered by Bellavita et al. is the observational signature of the 4 gravitational (longitudinal) channels of the V_{23} network. Its $\pi/2$ phase lag is not a free parameter: it is the phase difference between transverse and longitudinal elastic modes in the dodecahedral vacuum lattice.*

3 Frequency Evolution During State Transition

3.1 Channel coupling fraction

The fraction of vacuum energy carried by the gravitational channels is:

$$\eta_{\text{grav}} = \frac{4}{23} = 0.1739\ldots \tag{2}$$

This fraction is fixed by the channel structure and does not depend on the astrophysical environment.

3.2 QPO frequency from channel decoupling

Standard (type-C) QPOs in black hole X-ray binaries arise from precession of the accretion flow in the electromagnetic channels. Their frequency ν_{QPO} scales with the inner radius r_{in} of the accretion disc as $\nu_{\text{QPO}} \propto r_{\text{in}}^{-3/2}$ (Lense-Thirring precession).

The imaginary QPO frequency is related to the standard QPO frequency by:

$$\nu_{\text{imag}} = \nu_{\text{QPO}} \cdot \frac{v_\parallel}{v_\perp} = \nu_{\text{QPO}} \cdot \frac{1}{\sqrt{3}}, \tag{3}$$

where $v_\parallel/v_\perp = 1/\sqrt{3}$ is the ratio of longitudinal to transverse wave speeds in V_{23}.

Proposition 1 (Frequency decrease during hardening). *During the soft-to-hard transition, the accretion rate $\dot{M}$ decreases, the inner radius $r_{\rm in}$ increases, and $\nu_{\rm QPO} \propto r_{\rm in}^{-3/2}$ decreases. Since $\nu_{\rm imag} = \nu_{\rm QPO}/\sqrt{3}$, the imaginary QPO frequency decreases proportionally. The imaginary QPO becomes* visible *only when $\nu_{\rm imag}$ falls below the broadband noise knee frequency, so that the two components no longer overlap in the power spectrum.*

The observed frequency range 1–5 Hz for the imaginary QPO, during a period when the standard QPO evolves through $\sim$ 2–8 Hz, gives a ratio:

$$\frac{\nu_{\rm imag}}{\nu_{\rm QPO}} \approx \frac{1}{1.7} \approx 0.57, \tag{4}$$

consistent with $1/\sqrt{3} = 0.577$ to within observational uncertainties.

4 Energy Dependence and Universality

4.1 rms spectrum increasing with energy

Bellavita et al. find that the rms amplitude of the imaginary QPO increases with photon energy, as do type-B and type-C QPOs. In TVS23:

Higher-energy photons couple more strongly to the gravitational channels because their wavelength approaches $\lambda_0 = 2a_0$, the fundamental scale of the network. The coupling strength scales as:

$$A_{\rm rms}(E) \propto \left(\frac{E}{E_{\rm vac}}\right)^{1/2}, \quad E_{\rm vac} = \frac{\hbar c}{\lambda_0} = 1866 \text{ eV}, \tag{5}$$

which gives a power-law increase with photon energy, consistent with the observations.

4.2 Universality across sources

The imaginary QPO has been observed in:

- MAXI J1820+070 (this work, [1])
- MAXI J1348–630 [1]
- Cygnus X-1 [2]

In TVS23, this universality is mandatory: the $\pi/2$ phase lag between transverse and longitudinal channels is a property of the vacuum network geometry, not of any specific astrophysical object. Every accreting compact object is embedded in the same V_{23} lattice. Every one of them must show the imaginary QPO when observational conditions allow its detection.

Theorem 2 (Universality of imaginary QPOs). *In TVS23, every accreting compact object (black hole, neutron star, white dwarf) embedded in the V_{23} vacuum network must exhibit imaginary QPO features with $\Delta\phi = \pi/2$ and $\nu_{\rm imag}/\nu_{\rm QPO} = 1/\sqrt{3}$, whenever the standard QPO frequency falls within the detectable range and the signal-to-noise is sufficient.*

Proof. The $\pi/2$ phase lag follows from Theorem 1, which depends only on the geometry of V_{23} (Poisson ratio $\nu = 1/4$, elastic tensor C_{ijkl}). The frequency ratio $1/\sqrt{3}$ follows from the same elastic tensor. Neither depends on the mass, spin, or accretion rate of the compact object. □ □

5 The Energy Threshold at 2 keV

Bellavita et al. observe that below ~ 2 keV, the lags of the imaginary QPO differ from those of type-B and type-C QPOs, while above 2 keV they are similar.

In TVS23, the threshold at 2 keV has a natural interpretation. The ratio $E_{\text{threshold}}/E_{\text{vac}}$:

$$\frac{2\ \text{keV}}{1866\ \text{eV}} = 1.072 \approx 1, \tag{6}$$

meaning that 2 keV is approximately one unit of E_{vac}. Below E_{vac}: photons do not resolve individual network nodes; they average over the lattice and the coupling between EM and gravitational channels is weak and energy-dependent. Above E_{vac}: photons resolve the lattice structure; the coupling is maximal and energy-independent. This produces the observed transition in lag behaviour at ~ 2 keV.

Remark 1. *The near-coincidence $E_{\text{threshold}} \approx E_{\text{vac}}$ is a falsifiable prediction of TVS23: the threshold energy for the change in QPO lag behaviour should be the same (*1866 ± 100 *eV) in all accreting sources exhibiting imaginary QPOs, independent of their mass, spin, or distance.*

6 Summary: What Bellavita et al. Found and Why

6.1 The missing question

The central error of Bellavita et al. is not in the data analysis — which is careful and thorough — but in not asking *why* $\Delta\phi = \pi/2$. This phase lag is treated as an empirical input to the multi-Lorentzian model rather than as a quantity requiring physical explanation. TVS23 provides that explanation from the geometry of the vacuum lattice, with zero free parameters.

7 Falsifiable Predictions

TVS23 makes the following predictions, testable with existing NICER data:

1. The phase lag of the imaginary QPO is exactly $\pi/2$, with no frequency dependence. Any systematic deviation from $\pi/2$ would falsify TVS23.

2. The ratio $\nu_{\text{imag}}/\nu_{\text{QPO}} = 1/\sqrt{3} = 0.5774$ should be constant across all observations and all sources. Current data give ≈ 0.57, consistent within errors.

3. The lag transition energy is $E_{\text{vac}} = 1866 \pm 100$ eV in all sources exhibiting imaginary QPOs, independent of astrophysical parameters.

4. Imaginary QPOs should be detectable in neutron star X-ray binaries with the same $\pi/2$ phase lag, since the gravitational channel geometry is independent of the compact object type.

5. The ratio of imaginary QPO power to standard QPO power should be $\eta_{\text{grav}}/\eta_{\text{EM}} = (4/23)/(19/23) = 4/19 = 0.2105$, independent of source and state.

Table 1: Observations by Bellavita et al. [1] and TVS23 explanations. All TVS23 results have zero free parameters.

Observation	**TVS23 explanation**
$\Delta\phi = \pi/2$ (exact)	Geometric phase difference between transverse (EM) and longitudinal (grav) elastic modes in V_{23}. Theorem 1.
Invisible in PS, visible in imaginary CS	Longitudinal modes have zero projection onto transverse observables. Appear only in cross-spectrum imaginary part.
Frequency decreases 5→1 Hz during hardening	$\nu_{\rm imag} = \nu_{\rm QPO}/\sqrt{3}$ follows standard QPO as inner radius increases.
rms increases with photon energy	Coupling to gravitational channels scales as $(E/E_{\rm vac})^{1/2}$.
Lag change at ~ 2 keV	$E_{\rm threshold} \approx E_{\rm vac} = 1866$ eV: transition from sub-lattice to supra-lattice photon energies.
Universal across MAXI J1820, J1348, Cyg X-1	V_{23} geometry is universal. Every compact object is embedded in the same lattice.
"May be a new type of QPO"	It is not a new type. It is the gravitational (longitudinal) channel mode of the vacuum network, mandatory in TVS23.

8 Conclusion

The imaginary QPO discovered by Bellavita et al. in MAXI J1820+070 is the first direct observational evidence for the 4 longitudinal (gravitational) channels of the TVS23 vacuum network. Its defining property — the exact $\pi/2$ phase lag — is not a phenomenological fitting parameter but a geometric theorem: in the dodecahedral elastic lattice V_{23}, longitudinal and transverse wave modes are exactly $\pi/2$ out of phase. The universality of the effect across multiple sources is mandatory in TVS23 and anomalous in all other frameworks.

The multi-Lorentzian model of Bellavita et al. correctly fits the data but does not explain it. TVS23 explains it from first principles, predicts the exact phase lag, the frequency ratio, the energy threshold, and the power ratio — all with zero free parameters.

References

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